Int. J. Heat Mass Transfer. Vol. 4. pp. 31–45. Pergamon Press 1961. Printed in Great Britain.

BOILING HEAT TRANSFER

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(*Received* 23 *July* 1961)

Аннотация—Доклад посвящён проблеме теплообмена при пузырьковом кипении.

Сложность гидродинамических процессов в газожидкостных системах при пузырьковом кипении не дали до сих пор возможности построить полную теорию этого процесса.

В данной работе, исходя из гипотезы о гидродинамической природе кризисов в механизме кипения, сформулирована система дифференциальных уравнений, описывающих процесс пузырькового кипения, и составлена рациональная система характеризующих его критериев.

На основе анализа некоторых принципиальных вопросов теории теплообмена при кипении и составленной системы гидродинамических и тепловых критериев, характеризующих этот процесс, произведено обобщение экспериментальных работ в области пузырькового кипения.

NOMENCLATURE

c,	specific heat capacity;
$D = 2R$,	diameter;
g,	acceleration of gravity;
Δ_i,	difference between mean enthalpy of the flow and enthalpy of the saturated liquid;
L,	length;
p,	pressure;
q,	density of heat flow;
q_{cr},	critical density of heat flow;
r,	latent heat of vapour formation;
T,	temperature;
T'',	saturation temperature above a plane;
w_0,	circulation rate;
x,	gravimetric vapour content of a flow;
α,	heat transfer coefficient;
β,	volumetric vapour content of a flow;
γ,	specific weight;
δ,	transversal linear dimension of a channel;
ϑ,	underheating of the flow nucleus up to the saturation temperature;
λ,	thermal conductivity;
ν,	coefficient of kinematic viscosity;
σ,	coefficient of surface tension;
τ,	time.

Indices

$''$,	vapour phase;
int,	interface.

1. INTRODUCTION

BOILING heat transfer plays an essential part in solving a number of new problems and is widely used in the power industry. However in these cases the levels of formation of heat transfer surfaces can differ by some orders of magnitude. Thus, for example, when freon boils the heat fluxes are 10^3 kcal/m²/h in refrigeration technique, 10^4 in evaporators of electric power stations, 10^6 in water-moderated water-cooled reactors and 10^7 kcal/m²/h in elements of jet engines.

In connexion with this the value of critical density of heat flux is of decisive importance for reactors and other highly forced installations: for this value determines the highest limit of safe working of an assembly.

In industrial power installations the absolute values of heat transfer coefficients are of great importance, since they are usually of the same order of magnitude both for heating and heated media.

Film boiling takes place in hardening processes (heat treatment), at near critical pressures, and in some other specific cases.

The present paper deals with the problems of heat transfer with nucleate boiling.

Nucleate boiling is characterized by the origin of vapour in various places on the heating surface called nuclei of vapour formation. It creates a rather complex heterogeneous structure of liquid and vapour phase. This phenomenon relates to one of the most complicated problems of the hydrodynamics of gas–liquid systems, in particular to flows in which phase components of the flow are split up in separate units bounded by moving interface surfaces. The number of these units (bubbles, drops, films) variable in space and time is great, and therefore probability laws of multicomponent systems must be valid here.

At present there are no consistent analytical methods for such systems.

The purpose of the present work is to discuss some fundamental problems of the theory of heat transfer with boiling, and to generalize the results of experimental work.

2. ORIGIN OF VAPOUR BUBBLES

The intensity of heat transfer with nucleate boiling is conditioned by the interaction of convection, connected with the directed motion of the whole flow of a liquid, and local convection, caused by the origin, growth and rise of vapour bubbles. The latter process is of decisive importance at sufficiently high flux densities, i.e. at considerable rates of vapour formation.

The probability of the creation of a vapour phase in a superheated liquid is proportional to the number e raised to the power $1/-(\Delta t)^2$. Consequently vapour bubbles should rise just near the heating surface where the superheating of the liquid is the highest in comparison with the saturation temperature above the surface.

In some works on heat transfer with boiling an erroneous viewpoint of Kruzhilin [21, 22] has prevailed, namely that vapour bubbles rise on the projections of roughness. At present it may be considered to be established that the centres of boiling on a solid surface are hollows, pores and concavities near the base of protruberances and roughness. Apparently for the first time this mechanism was pointed out by Andreev [5], who studied cylindrical pores. This mechanism was then studied in the works of Nesis [33], Courty and Foust [57]. The rise of a vapour bubble on a conical pore was studied by Courty and Foust [57], Bankoff [54] and others. A review of these works was made by Zuber [72].

Thus the process of boiling is conditioned not only by the probability of bubble rise at a given degree of superheating of the liquid, but also by the probability distribution of centres of vapour formation on the heating surface. The probability character of the rise of vapour bubbles on a solid surface is proved distinctly by the curves of bubble distribution in accordance with the frequencies of their formation, which were obtained at certain pressures by Zysina and Kutateladze [15]. Similar curves were later obtained by Treshchev [48] for boiling by heating the nucleus of a liquid flow to the saturation temperature. The diameter of bubbles just leaving the heating surface is also not a constant value, but obeys some distribution law. Under static conditions the diameter of a bubble just leaving a smooth heating surface is determined according to Fritz [60] by equation:

$$D_0 = 0{\cdot}02\theta \sqrt{\left(\frac{\sigma}{\gamma - \gamma''}\right)}, \tag{1}$$

see [23].

The variability of D_0 points to the fact that the value of the marginal angle θ^0 cannot be considered as a distinct characteristic of a large area of a heating surface. Moreover recent cone-filming carried out at the Institute of Thermal Physics showed that when the boiling is intense a bubble has a complex form before breaking away, and that often it is bound to the heating surface by a cylindrical neck.

The facts cited above show distinctly that the hydrodynamic and heat processes going on with bubble boiling can only be statistically represented.

3. VAPOUR BUBBLE GROWTH RATE

The growth of a vapour bubble is connected with the density of the heat flow on a bubble surface by the equation

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = \frac{q_{\mathrm{int}}}{r\gamma''}. \tag{2}$$

Bošnjaković [55] applied this relation for determining the value q_{int} in accordance with the experimental data of Jacob and Linkew [64] and Jacob [65].

If a liquid medium round a bubble is regarded as stationary, then

$$q_{\text{int}} = C(T - T_{\text{int}}) \sqrt{\left(\frac{\lambda c \gamma}{\tau}\right)}, \qquad (3)$$

where the proportionality factor C is equal to $\sqrt{3}$ according to Plesset and Zwick [66] and to $-\pi/2$, i.e. to $C \approx 1{\cdot}6$ according to Forster and Zuber [59]; T is the temperature of a superheated liquid, and T''_{int} is the saturation temperature at the boundary of a bubble.

According to this formula, for the conditions of formation of one of the vapour bubbles photographed by the author and Zysina during the experiments, we have:

τ_{sec}	0·00475	0·0095	0·0285
q_{int} kcal/m²/h	203 000	143 000	83 000

Under the conditions of thermodynamic equilibrium for a spherical bubble

$$T''_{\text{int}} = T'' + \frac{2\sigma\gamma}{R(\gamma - \gamma'')} \cdot \frac{dT''}{dP}. \qquad (4)$$

While heat transfer is going on there is no thermodynamic equilibrium, a bubble grows due to liquid evaporation into it, and the value of T_{int} differs from that in equation (4). Assuming that there is only vapour in a bubble we may, in the first approximation, connect the rate of evaporation with the pressure drop in the vapour boundary layer by the known formula of the kinetic theory:

$$q_{\text{int}} = \xi r \Delta P \sqrt{\left(\frac{gm}{2\pi B T''}\right)}, \qquad (5)$$

where $\xi \leqslant 1$ is the accommodation coefficient, B is the universal gas constant, m is the molecular weight.

The corresponding change in saturation temperature is

$$\Delta T''_{\xi} = \frac{q_{\text{int}}}{\xi r} \sqrt{\left(\frac{2\pi B T''}{gm}\right)} \cdot \frac{dT''}{dP}. \qquad (6)$$

Summing up equation (4) and (6) we find that

$$T''_{\text{int}} = T + \left[\frac{2\sigma\gamma}{R(\gamma - \gamma'')} + \frac{q_{\text{int}}}{\xi r} \sqrt{\left(\frac{2\pi B T''}{gm}\right)}\right] \frac{dT''}{dP}, \qquad (7)$$

where

$$\frac{dT}{dP} = \frac{AT''(\gamma - \gamma'')}{r\gamma\gamma''}.$$

In technical units $A = \dfrac{1}{427}$.

The comparison of terms in square brackets of equation (7) is given in Fig. 1. It is distinctly seen from this figure that even at $\xi = 1$ the correction for the evaporation kinetics is not only commensurable with the correction for the curvature of the interface but in a number of cases it may exceed the latter.

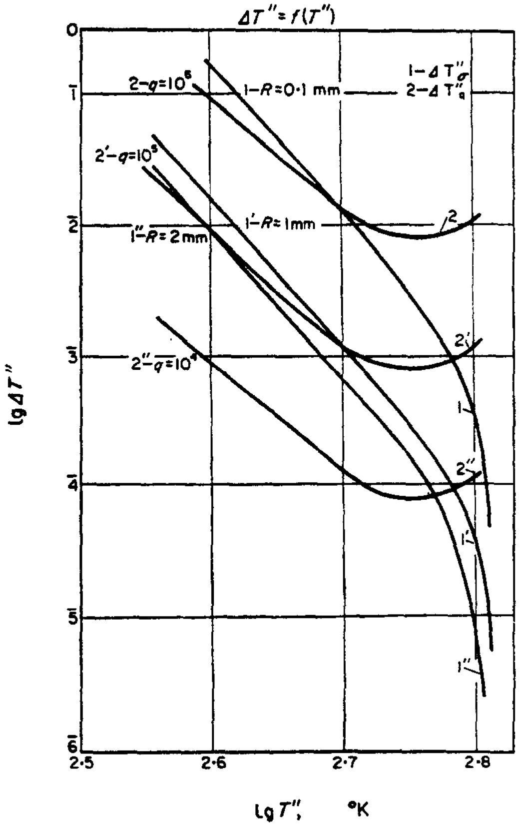

FIG. 1. Values of $\Delta T''_{\sigma}$ and $\Delta T''_{\xi}$ by equation (7).

4. FIRST CRITICAL HEAT FLUX DENSITY IN PIPES AND CHANNELS

It is usual to begin the study of boiling processes with the problem of free convection in a large volume of a liquid. From the technical standpoint, however, the most essential thing is the forced flow of a boiling liquid in tubes and channels.

At present vast experimental material has accumulated on critical heat flows in tubes and to a lesser degree, on critical heat flows in slotted channels.

Elucidation of the influence of pulsations on the value q_{cr} was of great importance. The number of regimes and thermal parameters determining the maximum heat flux density at which the nucleate boiling and, accordingly, a high intensity of heat transfer are still maintained is very high. In connexion with this the method of a generalized analysis of the phenomenon in question and the rational forms of connexion between the values determining it are of paramount importance.

Two viewpoints concerning the critical boiling in a large volume of a liquid were put forward.

According to the first viewpoint proposed by the author [24, 25] the transition from nucleate to film boiling is considered to be a qualitatively specific process, the basis of which involves the loss of the stability of structure of a two-phase boundary layer which had existed before the onset of the crisis.

Every critical hydrodynamic condition, as in this case the transition from one regime of boiling to the other, is determined not by a strictly fixed quantity of a characteristic value (q_{cr} in the present case) but only by a most probable value.

The second viewpoint most lucidly formulated by Kruzhilin [21], is based on the assumption that no new qualitative pecularities whatever arise during the first crisis in the boiling regime and, consequently, the critical heat flux is determined by the same initial sum of the generalized parameters (similarity criteria) by which the coefficient of heat transfer with bubble boiling is determined. This hypothesis was not developed for a forced flow but when applied to boiling with free convection, as shown in [28], it led to a very complicated empirical formula.

According to the hydrodynamic hypothesis the transition from nucleate to film boiling begins with loss of structural stability of a two-phase boundary layer in one of the small regions of the heating surface and from the formation of a steady vapour "spot". The rise of temperature under this "spot" caused by the drop of the heat transfer coefficient leads to a further development and distribution of film boiling over the heating surface. Such a concept fully corresponds to the phenomenon observed with boiling on heat resisting (e.g. graphite) rods and plates.

There is no basic difficulty in writing the conditions of mechanical interaction between a vapour and liquid for a differential element of the interface of such a system. And as far as the local stability of the structure is concerned the question of boundary conditions for the whole system does not arise. Accordingly, the system of equations consists of the equations of motion and continuity for each of the phases and for conditions of mechanical interaction of phases on the interface [25].

Considering, in the limit, a non-viscous liquid it is also possible to use the mechanism of the stability theory of Rayleigh's two-layer flow [39].

With such an interpretation of the problem it is the critical velocity of the vapour which has to be determined, and the question arises of the relationship between the local quantity of this value and the average heat flux on the heating surface.

It is evident that the only real measure of vapour formation rate is the value

$$\frac{q}{r\gamma''}, \tag{8}$$

representing the maximum vapour volume which can be formed per unit time on unit heating surface.

We consider here the maximum vapour volume since in a general case a fraction of the heat flow q may not be spent on vapour formation (e.g. in the flow where the nucleus is underheated to the saturation temperature).

From the heat balance of the mixture one more characteristic appears which can be presented in the form of the criterion:

$$K = \frac{r}{\Delta i} \tag{9}$$

where Δi is the difference between the mean flow enthalpy and the enthalpy of the saturated liquid.

In the flow of saturated liquid $1/K = x$ where x is the vapour content by weight, i.e. in this case the criterion K actually has a purely hydrodynamic meaning.

When $\bar{T} < T''$, then the criterion $K = r/c\vartheta$ where $\vartheta = T'' - \bar{T}$ is the underheating of the flow nucleus to the saturation temperature. In this case K has a purely hydrodynamic meaning only with fully turbulent heat transfer and serves as a measure of the vapour flow decrease due to the condensation process in a cool nucleus. If the molecular heat conduction is important, then K as the heat criterion is connected with the Prandtl number. This question will not be considered here.

The stability of the two-phase system is always affected by the dynamic head of the phase which is being produced, the scale of which in the given case is the value

$$\left(\frac{q}{r\gamma''}\right)^2 \frac{\gamma''}{q}, \tag{10}$$

as well as gravity and surface tension. As the scale of the joint action of the two latter forces [taking into account equation (1)] the following value can be taken

$$(\gamma - \gamma'') \sqrt{\left(\frac{\sigma}{\gamma - \gamma''}\right)}. \tag{11}$$

The square root of the ratio of these values gives the criterion [24, 25]

$$K = \frac{q}{r\sqrt{(q\gamma'')}\,\sqrt[4]{[\sigma(\gamma - \gamma'')]}}. \tag{12}$$

At $q = q_{\mathrm{cr}}$ the criterion of equation (12) can be determined.

If we use the Rayleigh stability criterion then the criterion K will be of the form

$$K = \frac{q_{\mathrm{cr}}}{r\sqrt{\left[q\gamma''\left(1 + \frac{\gamma''}{\gamma}\right)\right]}\,\sqrt[4]{[\sigma(\gamma - \gamma'')]}}. \tag{13}$$

The value of this expression is marked with the index 1 for the first critical density of the heat flow (the rise of film boiling) and with the index 2 for the second critical density of the heat flow (the destruction of the vapour film).

The difference between equations (12) and (13) by the value $\sqrt{[1 + (\gamma''/\gamma)]}$ is not so essential, and is connected with the solution of the initial system of equations adopted by Rayleigh.

It was shown in the works of the author [24, 25, 26] that in the range of the assumptions considered, the following relation exists between the dimensionless criteria:

$$K = \Phi\left[w\left(\frac{\gamma - \gamma''}{g^2\sigma}\right)^{1/4};\ \frac{\nu^2}{g}\left(\frac{\gamma - \gamma''}{\sigma}\right)^{3/2};\ \frac{\gamma}{\gamma''};\ \delta\left(\frac{\gamma - \gamma''}{\sigma}\right)^{1/2}\frac{L}{\delta};\ \frac{\Delta i}{r}\right]. \tag{14}$$

Moreover, the law of heat supply distribution along the heating surface should be predicted.

For the underheated liquid the hydrodynamic hypothesis led to the conclusion that there is a linear relation (at any rate in the first approximation) between the criteria K and $\Delta i/r$ of the form:

$$\frac{q_{\mathrm{cr},\vartheta}}{q_{\mathrm{cr},0}} = 1 - B\left(\frac{\gamma}{\gamma''}\right)^n \frac{\Delta i}{r}. \tag{15}$$

At a purely turbulent exchange the coefficient B is a certain constant. In the case of the influence of molecular heat conduction on heat transfer near the freshly forming elements of a vapour phase [e.g. by equation (3)] the coefficient B is the function of *Pr* and *Ar*.

As regards the experimental data we have to note that in spite of their multiplicity the agreement between the results of various investigations is still unsatisfactory. This criticism applies to qualitative results, apart from quantitative discrepancies which can be explained by the inherent instabilities of critical constants.

A number of discrepancies were explained after the role of the initial volumes of vapour bubbles in the rise of pulsations and the influence of the length of the heated section were revealed [4, 11, 46].

Most experiments with slotted channels and tubes show that the influence of the slot width or diameter is small at:

$$\delta > (1 \div 3) \sqrt{\left(\frac{\sigma}{\gamma - \gamma''}\right)}. \qquad (16)$$

The linear relation between q_{cr} and Δi in the region of underheating $\vartheta > 0$ is satisfactorily proved by the experimental data for sufficiently free channels at pressures up to the critical ones. Povarnin and Semenov [36] confirmed that the relation was satisfactory for liquid flow rates up to 40 m/s.

The experiments carried out by Zenkevich, Subbotin, Styrikovich, Miropolsky, Chirkin *et al.* [12, 13, 17, 46, 52] proved distinctly the general character of equation (15). However, the numerical values of the coefficients B and n are not equal in different experiments.

The experiments of Zenkevich [13], Miropolsky and Shitsman [17] give a clear dependence of q_{cr} on w_0 at $\vartheta \gg 0$ which is satisfactorily described by the empirical formula [24, 25]:

$$K_{10} = 0{\cdot}023\, w_0^{0{\cdot}5} \left(\frac{\gamma - \gamma''}{g^2 \sigma}\right)^{0{\cdot}125} \qquad (17)$$

where K_{10} is the value of the complex K at $\vartheta = 0$.

The experiments carried out by Chirkin and Yukin, Averin, Kruzhilin and Kutateladze with water flow in broad annular slots are also satisfactorily described by an equation of the type (17) but the proportionality factor is equal to 0·085 [25, 26]. The experiments of Ornatsky [35] are quantitatively 30 per cent lower.

The experiments of Doroshchuk and Frid [11, 17] show a weaker influence of the velocity of a liquid flow on the value of q_{cr}, and some influence of w_0 on the coefficient B is revealed. The experiments of Aladiev and Dodonov [3] differ in this respect from the results of other investigators. In these experiments the influence of w_0 on q_{cr} is practically zero at low pressures, while at high pressures it is comparatively weak and the monotonous relation between the coefficient $B(\gamma/\gamma'')^n$ and pressure is disturbed.

Some discussion on the relation between q_{cr} and w_0 at high velocities of a liquid flow is given in the work by Povarnin and Semenov [36].

In the range $x > 0$, i.e. in the flow of a saturated vapour–liquid mixture, the dependences for q_{cr} are still more complicated. It is positively established that the degree of influence of vapour content depends on the gravimetric mixture velocity or, in other words, on the so-called "circulation rate"

$$w_0 = \frac{G_{\text{mix}}}{\gamma \Omega}, \qquad (18)$$

where G_{mix} is the gravimetric mixture rate per second, Ω is the cross-section area of the channel.

When the pulsations are absent

$$\left(\frac{\partial q_{cr}}{\partial x}\right)_{w_0} < 0$$

i.e. the critical density of the heat flux decreases despite the essential rise in the average liquid flow rate at the increase in the vapour content of the mixture. Comparing this result with equation (17) it is necessary to conclude that the increase in vapour content of the flow essentially changes the vapour content of the layer near the wall, and favours the destruction of liquid structures existing in the layer.

On the basis of the experimental data available, more or less distinct relations may be outlined only for the region of volumetric vapour contents $\beta < 0{\cdot}9$. Since the volumetric and gravimetric vapour contents are connected by the formula

$$\frac{\beta}{1 - \beta} \cdot \frac{\gamma''}{\gamma} = \frac{x}{1 - x} \qquad (19)$$

then at low pressures very small weight vapour contents correspond to the values of $\beta < 0{\cdot}9$ while at high pressures they are of the order of 0·1 and more.

Rybin treated the experimental data obtained by Aladiev, Buchberg, Doroshchuk, Miropolsky, Subbotin and others on tubes of 9 mm more in diameter, vapour contents of $0 < \beta$ or $< 0{\cdot}9$ and entrance conditions under which the water entering the experimental section was slightly underheated to the saturation temperature. The results for the two values of the criterion:

$$F_* = w_0 \left(\frac{\gamma - \gamma''}{g^2 \sigma}\right)^{1/4} \qquad (20)$$

are given in Fig. 2 from which both the absolute discrepancies in the data of some investigators and the general scattering of experimental points are distinctly seen.

It is interesting to note that the average lines at the point $\beta = 0$ practically coincided with the results of calculations by equation (17).

In Fig. 3 the averaging straight lines are compared for various values of the criterion of equation (20). As is seen, the relative influence of vapour content weakens with the decrease of this criterion.

The experimental material available for the region of high weight vapour contents ($\beta > 0{\cdot}9$) is insufficient for generalizations.

Thus systematization and further accumulation of experimental data on flow in tubes and channels is essential before reliable calculation formulae for the value q_{cr} can be obtained. In connexion with this the complicated empirical

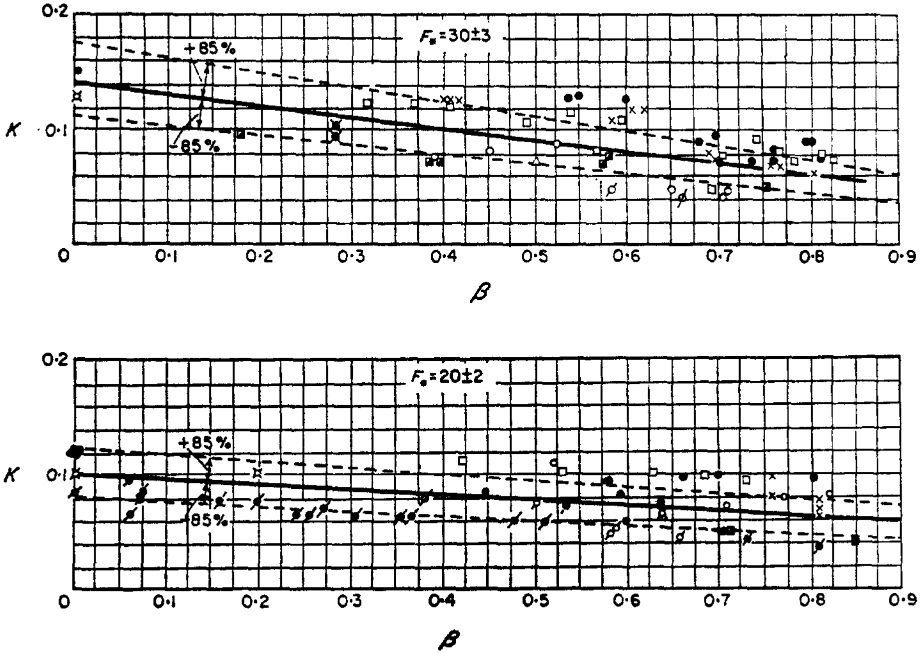

FIG. 2. Relation between K_1 on β and F_* in a flow in tubes.

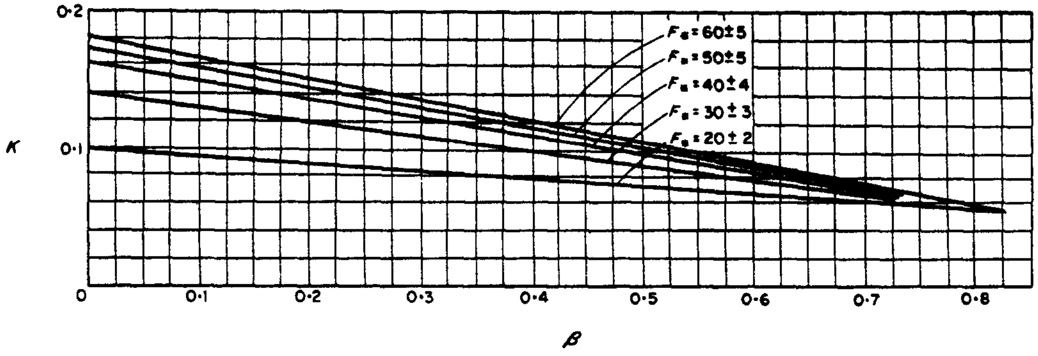

FIG. 3. Relation between K_1 on β and F_* $\beta < 0{\cdot}9$ (mean lines).

equations proposed by Zenkevich [13] and Ivashkevich [16] may be considered only as the first and not very successful experiment.

5. CRITICAL DENSITY OF HEAT FLOW AT FREE CONVECTION IN A LARGE VOLUME OF LIQUID

The results of the application of the hydrodynamic theory of control boiling to the conditions of free convection in a large volume of liquid are well known and reduce to the following expressions [24, 25]:

$$K_{10} \approx \text{const.}; \tag{21}$$

$$\frac{q_{\mathrm{cr},2}}{q_{\mathrm{cr},1}} < 1. \tag{22}$$

The influence of underheating of the bulk of a liquid is expressed by equation (15).

Using the Rayleigh stability theory of two-layer flow Zuber and Tribus [71] found that in non-viscous liquid

$$\left.\begin{array}{l} 0{\cdot}12 < K_{10} < 0{\cdot}16 \\ B = 5{\cdot}3\,(Pr^2\,Ar_*)^{-1/4} \\ n = \frac{3}{4} \end{array}\right\} \tag{23}$$

Equation (3) was also used for calculating the latter two coefficients.

It is interesting to note that the group B in equation (23) changes only very slightly. The theoretical values (23) coincide closely with the majority of the experimental data. However it is important to pay attention not to this good qualitative and quantitative corroboration of the theory by the majority of experimental investigations but to some discrepancies revealed. Apparently, if these deviations are not connected with experimental inaccuracy and inaccurate data on physical properties of the material investigated, they must serve as the starting point for further development of the theory. Here we mean small (less than 0·1) values of K_1 for melted salts revealed in experiments of Robin

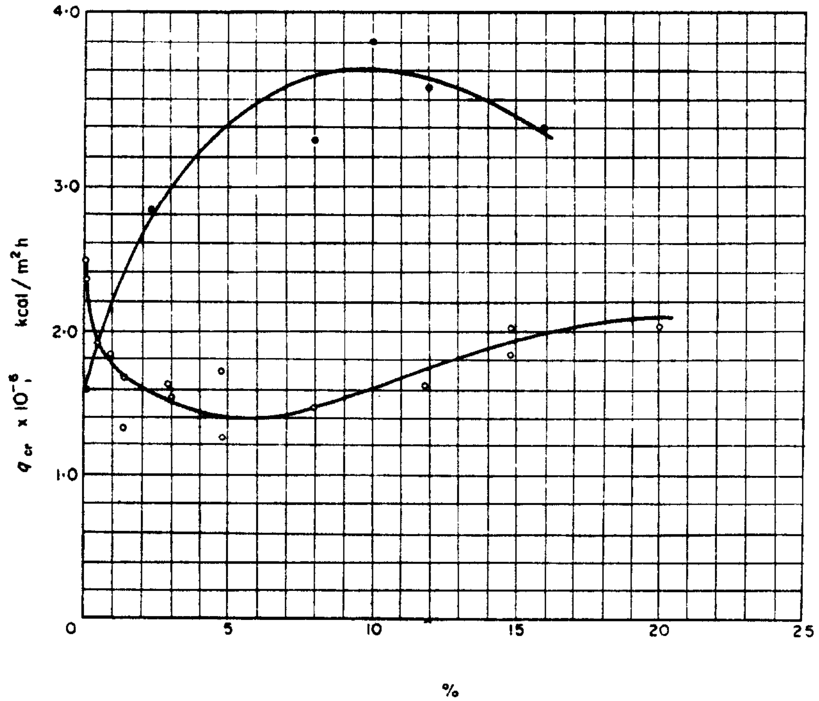

FIG. 4. Dependence of $q_{\mathrm{cr},1}$ on concentration of mixture of butyl alcohol in water:
●—wire, $p = 9{\cdot}8$ atm ○—plate, $p = 11$ atm.

[38] (with the approximate conservation of the values K_2) and the deviations of values B and n in the experiments for diphenyl from those for water and alcohol.

Highly interesting results in critical heat flows in mixtures with boiling on thin wires were obtained by Van Wijk, Van Stralen [69], Fastovsky and Artym [51]. It was found that the value of $q_{\mathrm{cr},1}$ essentially exceeded the corresponding values for each of the components at a certain composition of a binary mixture. For pure components the values of the critical heat flux were obtained coinciding well with equation (21) when a correction for viscosity was introduced according to Borishansky [6]

$$K_{10} = 0{\cdot}13 + 4\,Ar_*^{-0{\cdot}4}. \qquad (24)$$

The mechanism of this phenomenon is still far from clear. Moreover the experiments carried out by Bobrovich *et al.* in the Institute of Thermal Physics with heating surfaces greater than those used in the investigations mentioned did not discover this effect in any particular form (Fig. 4).

6. INFLUENCE OF FORCED CONVECTION ON HEAT TRANSFER WITH NUCLEATE BOILING IN TUBES

The influence of the directed convection characterized by the "density-displacement" velocity and vapour content of the flow on the intensity of heat transfer with nucleate boiling depends both on the density of the heat flow and the physical properties of the heat agent. Great attention is devoted to this problem in the works by Sterman, Styushin, Sorokin and others [18, 41–44].

With nucleate boiling the flow rate has little influence on the heat transfer coefficient at high pressure and powerful heat flow, as confirmed by the experiments of Aladiev, Tarasova *et al.* [2, 47].

At low pressures and moderate heat flows this influence may be quite essential.

As was shown by the author in [24, 29] the influence of circulation velocity on the heat transfer coefficient with bubble boiling can be estimated by the ratio of the heat transfer coefficient α_0 in the flow of a non-boiling liquid to α_{00} with a developed boiling.

For the region where there is no vapour content influence we have

$$\frac{\alpha}{\alpha_0} = f\left(\frac{\alpha_{00}}{\alpha_0}\right), \qquad (25)$$

and moreover

$$\left.\begin{aligned} &\frac{\alpha_{00}}{\alpha_0} = 0, \quad f = 1, \quad \dot{f} = 0; \\ &\frac{\alpha_{00}}{\alpha_0} \to \infty, \quad f \to \frac{\alpha_{00}}{\alpha_0}, \quad \dot{f} \to 1. \end{aligned}\right\} \qquad (26)$$

The simplest interpolation formula satisfying the conditions of (26) is

$$\frac{\alpha}{\alpha_0} = \sqrt[n]{\left[1 + \left(\frac{\alpha_{00}}{\alpha_0}\right)^n\right]}. \qquad (27)$$

The dependence of α on w_0 and q is shown in Fig. 5 according to test runs carried out by Sterman. It is distinctly seen from this figure that the influence of the circulation rate decreases with the increase in q. The treatment of these test runs shows that the value $n = 2$ convenient for practical calculations can be adopted [29].

The value α_{00} representing the heat transfer coefficient in developed boiling is determined for the given liquid by the equation:

$$\alpha_{00} = c(p)q^{0{\cdot}7}. \qquad (28)$$

For the experiments with water boiling in tubes the values of the pressure function $c(p)$ is shown in Fig. 6 according to the data by Aladiev, Sterman *et al.*, Tarasova *et al.* [2, 44, 47]. This dependence is qualitatively similar to that obtained by Borishansky [7] in experiments with boiling at free convection in a large volume of liquid but it is displaced somewhat lower. This quantitative discrepancy of some 20–30 per cent, is hardly fundamental in character.

The considered expressions are valid with $\beta < 0{\cdot}7$. It should be noted that the experimental material available is not plentiful for these values of vapour contents. The data available for the region $\beta > 0{\cdot}7$ are insufficient for any generalization to be made.

The experiments of Tarasova *et al.* [47] carried

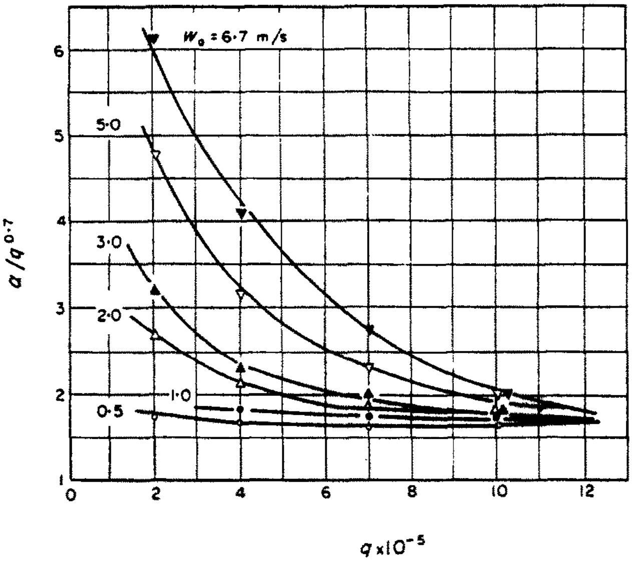

FIG. 5. Relation between $\alpha/q^{0{\cdot}7}$ and q and w_0 for water, $p = 2$ atm, tube 16 mm in diameter.

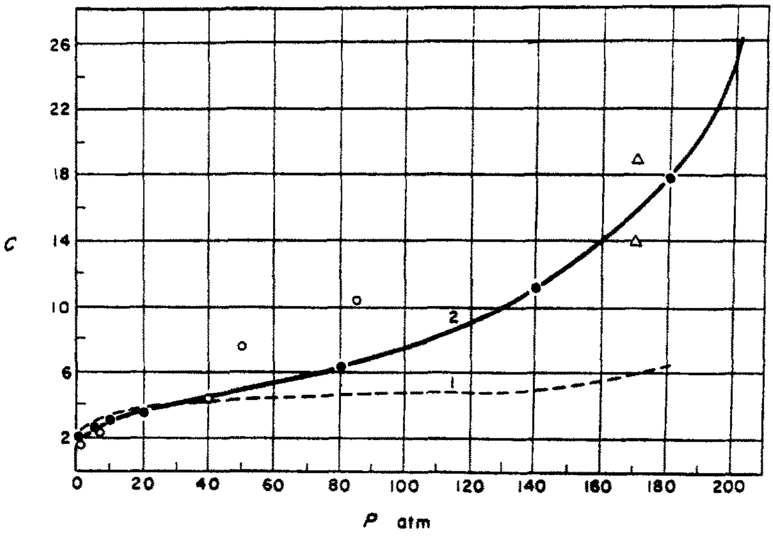

FIG. 6. The value of c_p in formula (28) from experiments with water boiling in tubes:

● according to [2]
○ according to [44]
△ according to [47]

Curve 1 by the Kruzhilin formula.

Curve 2 by the Kutateladze formula. In these two formulas the proportionality factors are diminished by 20 per cent.

out at 170 atm showed that at $\beta > 0{\cdot}7$ some increase at the beginning and then strong decrease of the heat transfer coefficient was observed.

7. HEAT TRANSFER WITH DEVELOPED NUCLEATE BOILING

The absence of any influence of velocity and channel sizes on the heat transfer coefficient with developed nucleate boiling (at least at $\beta < 0{\cdot}7$ and $D > 3\sqrt{[\sigma/\gamma - \gamma'']}$) is connected with the fact that in this case the sweeping of the heating surface by the liquid is fully determined by the process of vapour formation. This is another confirmation of the qualitative difference between the mechanisms of heat transfer with nucleate boiling and the appearance of film boiling, since the value q_{cr} depends quite distinctly on the velocity whereas α hardly depends on it.

Thus with developed boiling the heat flux q alone is the regime factor with respect to the heat transfer coefficient. Moreover there is the influence of the complex of physical properties of the medium varying along the saturation line.

The probability character of the distribution of the vapour formation centres over the heating surface and of the vapour bubble generation process itself creates the instability of the hydrodynamic and heat conditions in the region close to the wall as already mentioned above.

In connexion with this the values of heat transfer coefficients are far from being equal despite various experiments which were adequate in themselves. Due attention is not paid to this circumstance and some investigators still only consider their own values of α to be reliable.

When determining the similarity criteria of the heat transfer process it is necessary to study the interaction of the whole flow with the heating surface, i.e. one should not limit oneself to the investigation of local conditions, as was the case in the problem of the critical regime. This circumstance creates some difficulties, not yet overcome, for formulating the corresponding system of equations and in addition to them, the conditions on the boundary of the system.

However, these difficulties differ slightly from similar ones which occur with the analytic formulation of conventional turbulent flow, which also consists of a statistical sum of finite formations, variable in space and time.

Since at boiling there are parts of the medium with different physical properties (liquid and vapour), the mathematical formulation of the problem must contain the relations both for the phenomena inside each of the regions and for the interactions (mechanical and thermal) on their surfaces, i.e. at the interfaces.

The conditions on the boundaries of the whole system, i.e. on the heating surface should be predicted in an independent manner.

All the interactions taking place in a two-phase flow of any complexity are direct consequence of the total effect of local interactions. Therefore if we assume that this totality does not introduce any specific physical properties, the mechanism for the derivation of the similarity criteria is made up of the system of differential equations of motion and heat conduction in each phase, the conditions of local interactions on the interface and of the conditions on the heating surface (boundary conditions).

Essentially, this problem was raised in the same form by the author [23, 24] and later on by Eigenson [53].

This approach to the derivation of the similarity conditions can be methodically formulated in two ways:

(1) The equations of heat transfer and motion in a differential form for each phase, the equations of the interaction on a differential surface element of the interface and the system boundary conditions are written out;

(2) The equations of heat transfer and motion in the differential form are written out for the phase which is of decisive importance for the process (for the liquid in nucleate boiling); the second phase is considered in the form of finite elements of definite geometry (e.g. spherical vapour bubbles) the equations of motion of the centre of gravity of such an element and heat balance equations are written out, the system boundary conditions being the same as in the first case.

In a like manner the system of equations can be made up when considering a new phase to be a heat and mass sink [10, 42]. Such a procedure does not differ in essence from the one above but in this case the discrete elements of the new phase should be considered as material points. Therefore such a method may be applied when the region where the process is going on is very large in comparison with the elements of the dispersed phase.

There is no basic difference between the two methods since the motion equation of the bubble centre of gravity can be obtained by solving the equation system of the complete motion of both phases. Such solutions were obtained in the finite form for some ranges of Reynolds numbers by Hadamard [61] Rybchinsky [68] and Levich [30].

Naturally the system of equations should correspond to the adopted model of the real physical process and be closed in the sense that the number of equations corresponds to the number of unknowns entering them. The degree of the model conformity with the real process is in the end given by the comparison between experimental data and the results of generalizations. This circumstance is not of course peculiar to the boiling process by has general significance [19].

The consideration of equations of liquid motion and heat conduction with regard to the equations of motion and mechanical and thermal interaction of a vapour phase gives three criteria:

$$Re_* = \frac{q}{r\gamma''\nu}\left(\frac{\sigma}{\gamma-\gamma''}\right)^{1/2};$$

$$Ar_* = \frac{g}{\nu^2}\left(\frac{\sigma}{\gamma-\gamma''}\right)^{3/2}\left(1-\frac{\gamma''}{\gamma}\right);$$

$$Pr = \frac{\nu}{Q}. \tag{29}$$

Here the value $q/r\gamma''$ is the velocity scale and the value $\sqrt{(\sigma/\gamma - \gamma'')}$ is the scale of the linear dimension.

At highly intensive boiling the layer near the wall may be so turbulized that the influence of the molecular viscosity may be neglected in the equation of motion. In such a case we obtain only two criteria as distinct from (29):

$$Pe_* = Pr\,Re_* = \frac{q}{r\gamma''Q}\left(\frac{\sigma}{\gamma-\gamma''}\right)^{1/2};$$

$$Pr^2 Ar_* = \frac{g}{Q^2}\left(\frac{\sigma}{\gamma-\gamma''}\right)^{3/2}\left(1-\frac{\gamma''}{\gamma}\right). \tag{30}$$

In particular, this is valid for the media with $Pr \ll 1$. It follows from boundary conditions that:

$$Nu_* = \frac{\alpha}{\lambda}\left(\frac{\sigma}{\gamma-\gamma''}\right)^{1/2}. \tag{31}$$

The last criterion was introduced by Jacob [65].

The continuity condition of the temperature field on the interfaces has the form:

$$T_{\text{int}_1} = T_{\text{int}_2} = T''_{\text{int}}. \tag{32}$$

If T''_{int} is determined by equation (4), as Kruzhilin determined it [21], then a new criterion is obtained which can be reduced to the form:

$$K_t = \frac{(r\gamma'')^2}{Ac\gamma T''\,\sqrt{[\sigma(\gamma-\gamma'')]}}. \tag{33}$$

If the kinetics of evaporation is taken into account as was done by Zysina and Kutateladze [14, 24], the following criterion appears:

$$K_p = \frac{p}{\sqrt{[\sigma(\gamma-\gamma'')]}}. \tag{34}$$

As was shown above, the terms of the equation which lead to these criteria are at least commensurable.

If we assume that $\mathrm{d}p/\mathrm{d}T'' \approx p/T''$ then:

$$\frac{K_t}{K_p} \approx K_s\frac{\gamma''}{\gamma}\left(1-\frac{\gamma''}{\gamma}\right) \tag{35}$$

where:

$$K_s = \frac{r}{cT''} \tag{36}$$

is the modification of the criterion K introduced by Sterman.

The treatment of experimental data with simultaneous use of the criteria K_t and K_p (or K_p and $K_s\,\gamma''/\gamma$) is difficult. However it turns out that there is nearly a single-valued relation between these criteria for many liquids. And this explains the fact that the empirical formulae built by the limited system of criteria as:

$$Nu_* = f_1(Pr;\quad Re_*;\quad K_p)$$

$$Nu_* = f_2(Pr;\quad Re_*;\quad K_t), \tag{37}$$

give more or less satisfactory results.

In practice for a particular liquid one treatment is somewhat preferable.

Tobilevich [49] and some other authors showed that the treatment with the criterion K_p gives slightly better results especially in the region of high pressures. In particular this refers to water (Fig. 6).

Some authors striving for better agreement between experimental data developed formulae which contain a limited number of criteria and individual constants for each liquid. These constants are even commensurable in some formulae. In this connexion we may mention the formulae of Rohsenow [67], Aladiev [2], Jacob and Linke [64], Tolubinsky [50] and Levy [62]. Apparently, in this case it is preferable to have simple empirical formulae for the given substance. Aladiev suggested a good form of such a formula for water. It is expedient to give this formula as:

$$\alpha = \frac{c_q^{0\cdot 7}}{1 - (2\cdot 56 \times 10^{-3} t'')}, \qquad (38)$$

where c is the constant characterizing the heating surface. With boiling in tubes $c \approx 2\cdot 1$; with boiling in a large volume on brass tubes $c \approx 2\cdot 6$ and $c \approx 3$ with boiling in a large volume on tubes of stainless steel.

In this respect the methods of developing empirical relations on the basis of the concepts on the states of the substance are useful. This subject is considered by Novikov [34], Borishansky [8], Rychkov [40] and others.

CONCLUSIONS

(1) The available theoretical and experimental data on bubble boiling mechanism concern, in the main, the conditions under which bubbles interact slightly with one another. With boiling of a saturated liquid at high thermal flux densities the development mechanism of vapour bubbles changes considerably. Further investigations are needed for creating a sufficiently complete theory of nucleate boiling.

(2) The hypothesis of hydrodynamic nature of crises in boiling mechanism and crisis development from local disturbances of the stability of a two-phase boundary layer allowed a strict formulation of the system of differential equations describing this process, the choice of both a rational form of the criterion determined and the system of principal determining criteria.

On the basis of this hypothesis theoretical formulas are obtained for the first critical heat flux in a large volume of liquid with free convection.

The qualitative conclusions from the hydrodynamic hypothesis are also confirmed well enough by the data on forced flow.

For further development of this problem it is necessary to carry on a number of widespread investigations of critical phenomena in the flow of liquids with different physical properties. It is also necessary to specify exactly the conditions of experiments.

It is especially necessary to accumulate experimental data in the region of positive flow vapour contents.

(3) The influence of circulation rate on the heat transfer coefficient with nucleate boiling depends on the ratio of intensities of heat transfer with conventional convection, and with developed boiling. In certain cases this influence may be quite essential and promote the operation of heat transfer equipment.

(4) The strict formulation of the problem of heat transfer intensity with bubble boiling is possible only on the basis of consideration of statistics of many final elements variable in space and time. At present such statistics are not available.

The ways of overcoming this difficulty discussed here in terms of deriving similarity conditions lead to an adequately complete system of hydrodynamic and thermal criteria. It is impossible to give strict formulation of the conditions on a heating surface.

(5) In practice the heat transfer conditions are not studied for boiling of binary mixtures or in other specific cases. The amount of work in this respect in no way corresponds to the practical and theoretical importance of this problem.

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Abstract—The present paper is devoted to the problem of nucleate boiling heat transfer.

Complicated hydrodynamic processes occurring in gas–liquid systems with nucleate boiling have made it impossible up to now to develop a complete theory for this process.

A system of differential equations for a nucleate boiling process has been formulated and a rational system of criteria characterizing it has been developed, proceeding from the hypothesis on the hydrodynamic nature of crises in the boiling mechanism.

Generalization of experimental works on nucleate boiling has been carried out on the basis of the analysis of some principal problems on the boiling heat transfer theory and a system of hydrodynamic and heat criteria characterizing this process.

Résumé—Cet article est consacré au problème de la transmission de chaleur au cours d'une ébullition nucléée.

Les phénomènes hydrodynamiques compliqués qui interviennent dans le système liquide–gaz lors d'une ébullition nucléée ont jusqu'à présent rendu impossible, l'établissement d'une théorie complète de ce processus.

Cet article établit un système d'équations différentielles relatif à une ébullition nucléée caractérisée par des critères rationnels fondés sùr l'hypothèse faite sur la nature hydrodynamique du mécanisme de l'ébullition.

La généralisation des travaux expérimentaux sur l'ébullition nucléée a été faite à partir de l'étude de quelques problèmes essentiels de la théorie de la transmission de chaleur par ébullition et d'un système de critères thermiques et hydrodynamiques caractéristiques de ce processus.

Zusammenfassung—Der Wärmeübergang beim Blasensieden ist hier behandelt. Die komplizierten hydrodynamischen Prozesse, die in Gas–Flüssigkeit-Systemen beim Blasensieden auftreten, machten es bis jetzt unmöglich, eine geschlossene Theorie für diesen Vorgang zu entwickeln. Hier wird ein System von Differentialgleichungen für den erwähnten Vorgang aufgestellt und durch eine Reihe von Kriterien charakterisiert, die auf der Annahme der hydrodynamischen Natur der burn-out-Erscheinungen beruhen. Auf Grund der Analyse einiger prinzipieller Probleme des Wärmeübergangs beim Sieden und aus einem System hydrodynamischer und wärmetechnischer Charakteristika des Vorgangs konnten Verallgemeinerungen für experimentelle Arbeiten getroffen werden.